*Modeling* for STEM Education Reform

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I. Modeling Theory of Cognition

II. Scientific Models & Modeling Tools

III. Modeling Instruction & Curriculum Design

IV. Empowering Teachers for STEM Education Reform

U. Oregon, May 2017

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I. Modeling Theory of Cognition

Basic questions about cognition:

- What is the difference between conceptual and rote knowledge?
- What is a concept?
- How do words get meaning?
- *"What, precisely, is thinking?" — Einstein*

Key to the answer: **Symbolic Forms** (coined by Cassirer)

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The *prototypical example* of cognition is **comprehension of a narrative**:

A narrative may be **read**, heard or observed
(as from clues read by a hunter in tracking animals)

This *generates construction of a mental model* in the subject, as schematized in the diagram:

Narrative structure (universal components):
- Characters
- Setting
- Events

correlates with

Mental Model
cognitive structure

**Crucial point:** comprehension involves creating or invoking a *correspondence between narrative structure and cognitive structure* that I call a **symbolic form**

Modeling theory generalizes this insight to other cognitive domains:

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**Modeling theory of cognition**

- **Imagination** • **Understanding** • **Representation**

Mental model ⟷ Symbolic construct

embodied structure ⟷ **morphism** ⟷ symbol structure

*meaning* ⟵ **concept** ⟶ *symbolic form*

- **Imagination** ≡ generates {**ideas**: mental models (including scenarios)}
- **Understanding** ≡ elevates **ideas to concepts** by creating symbolic forms to represent the ideas

**Meaning** ≡ **structure** (**form**) embodied in **mental models**

**Form** ≡ the structure of an *object*, *model* or *system* (real or imagined)

**Morphism** ≡ a form-preserving mapping (analogy)

Elucidating the concept of **Concept** ≡ a {form, meaning} pair

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A basic insight of **Cognitive Linguistics**:
**the referents of language are mental models!**
**Modeling Theory** refines this with the concept of **symbolic form!**

The **symbolic form** of a **concept** is a triad:

**CONCEPT**: *form* (abstract structure), *meaning* (private), *symbol* (public)

A **conceptual model** is composed of concepts linking mental structures to representations.

The **referent** of a conceptual model is a mental model.

**Conceptual Model**: *structure*, *referent*, *representation*

A similar distinction has been made for mathematics concepts:

**concept image** ⟷ **concept definition** (Tall & Vinner, 1981)

linked by symbolic form!

Slogan: **No conception without representation!**

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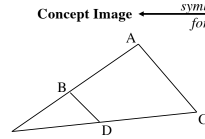
**Proportional reasoning is the gateway to mathematics literacy!**

**EUCLID**: *proportion as* ***analogy*** between *figure & number*

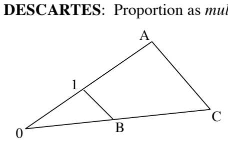
**Concept Image** ⟵ *symbolic form* ⟶ **Concept Definition**




A:B :: C:D

A/B = C/D

**DESCARTES**: Proportion as *multiplication of line segments*
→ analytic geometry




A/1 = C/B

C = AB

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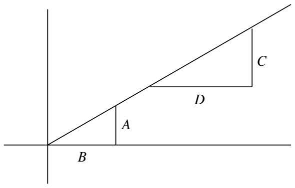
PROPORTIONS in the math curriculum today

Geometric figures → graphs → enables depiction of linear relations among ratios of quantities with **different units**

Equivalent **ratios** as proportions → **rates** as slopes → comparison of measurement pairs

**Concept Image** ←——→ **Concept Definition**




$$\frac{A}{B} = \frac{C}{D} = S$$

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What are the intuitive primitives that underlie proportional reasoning?

Here is one answer: Andy diSessa (1993) has identified irreducible conceptual primitives used in causal reasoning, which he calls *phenomenological primitives* (**p-prims** ). In particular:

**Ohm's p-prim** comprises "an agent that is the locus of an impetus that acts against a resistance to produce a result."

Provides the intuitive **causal syntax**:

agent → (kind of action) → on patient → result (concept image)

Serves as *intuitive structure* for $\mathbf{F} = m\mathbf{a}$ (concept definition)

Also provides intuitive **logical syntax** for:

a *qualitative proportion*: more effort ⇒ more result

and the *inverse proportion*: more resistance ⇒ less result

Caveat: The ***Force is Action*** metaphor

*must be adjusted for **consistency with Newtonian physics!***

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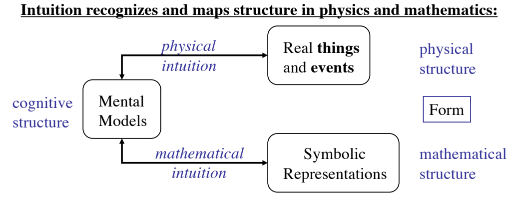
**Intuition recognizes and maps structure in physics and mathematics:**




**Physical intuition** ≡ matches mental models directly to physical systems

But the **referents of physics** theory are **conceptual** models of data, observations and experiments rather than physical objects & events.

**Mathematical intuition** ≡ matches mental models to symbolic structures

**Mathematical understanding** requires development of both *physical and mathematical intuition*, which supply the essential *repertoire of mental structures* for constructing math meaning

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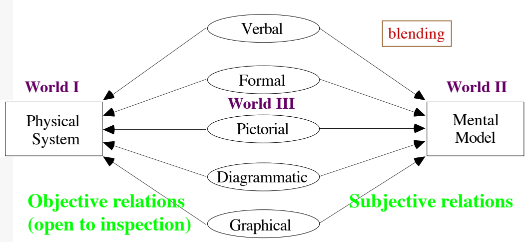
**Physical intuition** integrates structure in multimodal representations into a coherent mental model




**Mutual understanding requires negotiation of meaning**
to achieve intersubjective agreement
on the use of representations.

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**Modeling Tool Development**

| Instruments for detecting patterns | precision | Symbolic systems for representing patterns |
|---|---|---|
| Perceptual | Stage I | Cognitive |
| • Sensory apparatus (direct observation) | ↓ | • natural language • sketches & icons |
| Experimental | Stage II | Theoretical |
| • scientific instruments & apparatus | ↓ | • mathematics • diagrammatic systems |
| Computer | Stage III | Computer |
| • Universal Lab Interface | ↓ | • simulation • visualization |

**Instrumental tools extend perceptual powers**

**Symbolic tools extend cognitive powers**

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## Major conclusions from cognitive science

### I. Composition of mental models

***Mental models represent states of the world***, not perceptions
– are ***schematic***, representing only some features of things
– are ***structured***, consisting of ***elements and relations*** between them
– ***elements* are typically *objects*** (or reified things).
– **object *properties* are idealized** (points, lines or paths).

• Object models always *placed in a background* (context or ***frame***).

• Individual objects are *modeled separately* from the frame, so they can move around in the frame.

**Major conclusion**: *Structured external representations, such as language and mathematics* (**symbolic forms**), support construction of mental models.

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### II. Model-based reasoning

– is more general and powerful than propositional logic
– integrates multiple representations of information
(propositions, maps, diagrams, equations)
into a coherently structured *mental model*
(typically distributed across many representations)

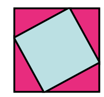
**Rules and procedures** define mathematical structure
– are central to modern concept of inference!
– can be understood as prescriptions for **operations on** mental models as well as on symbolic representations
– probably encoded in the motor system

**Deductive inference** from models or propositions

*Deduction is extraction of information from a structured object or system* (Barwise)

**Paradigm:** Inference from maps

*Inference by inspection:* **Behold!**



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### What is a model?

**A model** is a **representation of structure** in a system of objects

**Four types of structure** suffice for any scientific model
(a) **systemic structure** specifies
• ***composition, object properties*** and ***causal links***
(b) **geometric structure** specifies
• ***configuration*** and ***location*** in a ***reference frame***
(c) **interaction structure** specifies
• ***interaction laws*** for causal links
(d) **temporal structure** specifies
• ***change*** in state variables

Optimal precision in definition and analysis of structure is supplied by

**Mathematics**: ***the science of structure***

Different types of structure are characterized by
Set theory, Group theory, Geometry, Topology, Algebra

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MODEL SPECIFICATION (for structure in a physical system)

I. Organization: composition and connectivity (*topological structure*)
• (internal) constituents
• (external) agents — Represented by: **System schema** (Organization Chart)
• connections

II. Geometry (*geometric structure*) — Rep. by: Maps
• ***Size and shape*** variables: Volume V, Area A, length L, width
• ***Position*** (vector **x** for center of mass, distance, displacement) & ***orientation*** in a reference frame

III. Interactions (*interaction structure*: interaction laws)
• (internal) ***state variables***: e.g. $V, P, T, E$ = internal energy
• ***equations of state***: e.g. $PV = nRT$, $E = E_K + \ldots$ — phase diagrams
*or constitutive equations*: e.g. for rigidity, elasticity, magnetism
• **Force Laws** : e.g. $F = GmM/r^2$, Lorentz Force — **force diagrams**

IV. Processes (*temporal structure*: laws of change)
• Equations of motion & field equations
• Equations for change of phase
• Rates of chemical and nuclear reactions
• Thermodynamic change in **energy and entropy**
• **Conservation Laws**

Motion maps
Flow diagrams
Bar charts

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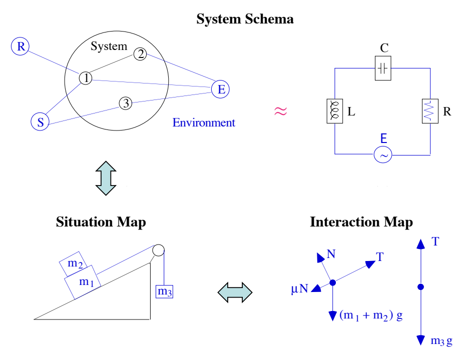
### Graphic Representations of Topological Structure

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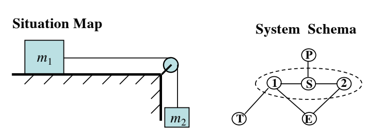
Systematic use of **system schemas** as a modeling tool
has the largest immediate impact on student performance
of any teaching innovation I know!! (Joe **Redish,** Maryland)

It tells students **how to start** solving any given problem!




**System schemas are not unique!!**

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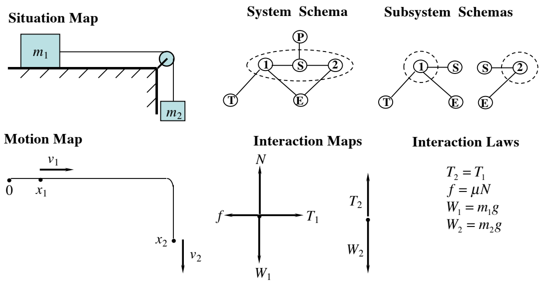
**Distributed representations of structure in a conceptual model**




**Geometric Structure**
$x_2 = x_1 + L$
➔ $v_2 = v_1$
➔ $a_2 = a_1$

**Temporal Structure:** (Eqns. of Motion)
$T_1 - \mu N = m_1 a_1$
$m_2 g - T_2 = m_2 a_2$
$N = m_1 g$
➔ $m_2 g - m_1 g = (m_1 + m_2) a_1$

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*Content core*:

**Basic Particle Models in Newtonian Mechanics**

| *Kinematical Models* | *Causal Models* |
|---|---|
| Constant velocity | Free Particle: $\Sigma \mathbf{F}_i = 0$ |
| Constant acceleration | Constant force: $\Sigma \mathbf{F}_i$ = constant |
| Simple Harmonic Oscillator (SHO) | Linear binding force: $\Sigma \mathbf{F}_i = -k\,\mathbf{r}$ |
| Uniform circular motion (UCM) | Central force (with constant $\lvert \mathbf{r} \rvert$) |
| Collision $\Delta \mathbf{p} = \mathbf{I}$ | Impulsive force |

**Instructional design:** spend two weeks developing each of these models in an instructional cycle that emulates scientific practice!

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All *measurements are comparisons* expressed as **ratios** or **rates**!! (per)

**Quantitative methods** *require mathematical models* to interpret data!

**Basic Mathematical Models** are **about RATE (of change)!**

1. **Constant rate** (linear change): graphs and equations for straight lines (proportional reasoning, constant velocity, acceleration, force, momentum, energy, etc.)
2. **Constant change in rate** (quadratic change) graphs and equations for parabolas (constant acceleration, kinetic and elastic potential energy, etc.)
3. **Rate proportional to amount**: doubling time, graphs and equations of exponential growth and decay (monetary interest, population growth, radioactive decay, etc.)
4. **Change in rate proportional to amount:** graphs and equations of trigonometric functions (waves and vibrations, harmonic oscillators, etc.)
5. **Sudden change**: stepwise graphs and inflection points (Impulsive force, etc.)

*Ubiquitous*: rich & unlimited applications to science and modern life!

*Skill in using these models* in a variety of situations
- an essential component of math and science literacy.
- should be *cultivated* deliberately, systematically & repeatedly

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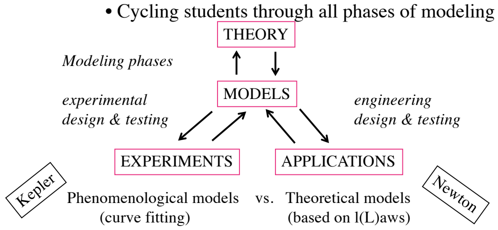
Modeling Instruction – in a nutshell!

A **science pedagogy and curriculum design** centered on
- Scientific models as the content core of each science
- Modeling as the procedural core of science
- Cycling students through all phases of modeling

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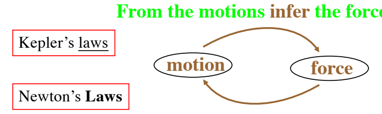
Systematic mathematical modeling began with Galileo and Newton

**Newton's Modeling Cycle:**

– *Preface to the Principia*

**From the motions infer the forces!**




**From the forces deduce the motions!**

Newtonian Theory specifies the rules for a **modeling game: (research program):**

**To find all the forces** necessary and sufficient to account for all observed motions in the universe!

Ref: *Modeling Games in the Newtonian World* (1992)

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### Why scientific practice is model-centered!?

I. Theoretical

- **Models are basic units of coherently structured knowledge**.
  from which one can make logical inferences:
  ***predictions, explanations, plans and designs***
- One cannot make inferences from isolated facts or theoretical principles
- **A model can serve as inferential tool** for the kind of structure it embodies.

II. Empirical

- Models can be **directly compared with physical things and processes**.
- A theoretical hypothesis or general principle cannot be tested empirically except through incorporation in a model

III. Cognitive

- Model structure is concretely **embodied in physical intuition**, where it serves as an element of *physical understanding*

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### Elements of Modeling Instruction

*Impediments to learning* physics:
(a) Misconceptions about common physical phenomena.
(b) Misconceptions about scientific method.
(c) A view of science as a fragmented collection of facts, rules and formulas.

*Instructional objectives* include:
(a) a clear *concept of "physical model,"* including both qualitative and quantitative aspects,
(b) familiarity with a *basic set of models* as the core of introductory physics,
(c) skills in the *techniques of modeling*, especially interplay between diagrammatic and symbolic representations,
(d) experience in the *deployment of models* to understand the physical world--to interpret and analyze data, to explain, to predict and to plan.

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### *Pedagogy versus Content*

The most robust finding of physics education research:
*You cannot separate pedagogy from content!!*

*Modeling Instruction* integrates curriculum and pedagogy:

- *Curriculum* is organized around a small number of conceptual models as the *content core* of each scientific domain.
- *Pedagogy* promotes scientific inquiry centered on making and using models as the *procedural core* of scientific knowledge
- Applicable to *all STEM disciplines!!*

- Example: abstract mathematical concepts such as *variable*, *function* and *rate* can be explored within the context of mathematical models with concrete applications in physics and deployed to other subjects (i.e. chemistry, biology, economics).

***Modeling Pedagogy*** **– flexible, evolving, research-based**

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### Modeling Instruction promotes Scientific Inquiry

I. Instructional design:

The instructional **modeling cycle**
engages students in all aspects of **scientific inquiry:**

- *Empirical*: Design and conduct experiments to investigate *structure* in physical systems and processes.
- *Theoretical*: Construct, analyze and apply *scientific models* and *theories*.
- *Technical*: Use scientific instruments and modeling tools to sharpen scientific investigation and inference.
- *Social*: Scientific discourse and argumentation to negotiate mutual understanding of models and implications of experimental results.

II. Teachers guide student inquiry by

- *organizing activities and discourse around scientific models*
- informed by research on student conceptual learning

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### Empowering Teachers

**for rapid, deep, and sustained STEM Education reform**

**HOW to do it:**

Reform will be
- **rapid** if it *focuses* on
  *empowering teachers already in the classroom*
- **deep** if it is *anchored* in
  discipline-based science education research
- **sustained** if it is *linked directly* to scientific communities for continual renewal and adaptation to the rapidly changing world of science and technology.

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**Three Institutions** needed to drive STEM reform:

**Rapid:** **I. AMTA**: a professional community of teachers that cultivates *Modeling Instruction* as a unified approach to quality STEM teaching & curriculum design.

**Deep:** **II. PER**: a professional community of practice dedicated to research on teaching and learning physics.
– Modeling Instruction is grounded in PER and requires PER for continuous improvement.

**Sustained:** **III. PhysTEC:** to organize a nationwide *partnership* of university/college physics departments to provide
(1) sites for the summer workshops,
(2) local support for STEM teachers,
(3) coordination with pre-service teacher preparation and induction programs.

**The failed alternative:** K-12 schools and school systems lack the necessary expertise and resources to reform STEM education.

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## Cultivating Teacher Expertise

- *Development of expertise in any domain requires ten years (or 10,000 hours) of **deliberate practice!****
- Years of classroom teaching experience will not improve teaching expertise!

**Professional preparation for teachers compared to scientists**

| | 4 yr. college | | Total prep. |
|---|---|---|---|
| Scientist: | 3 in major | 4-7 in grad school \| 2-3 as post-doc | ≈ **10 yrs.** |
| Teacher: | 2 in major | teaching begins, sporadic prof. dev. | ≈ **3 yrs.** |

Red numbers indicate years with strong opportunity for deliberate practice.

**Feasible solution:**
- Intensive summer multi-week workshops offered by the AMTA
- Lifelong professional development opportunities at universities and national labs like Brookhaven and Fermilab
- Graduate studies for teachers like the MNS program at ASU

*K. Ericsson, et.al. (1993) The Role of Deliberate Practice in the Acquisition of Expert Performance, *Psychological Review* **100**: 363-406.
*M. Gladwell (2008) *Outliers*. New York: Little, Brown and Company

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To **amplify teacher power:**

AMTA American Modeling Teachers Association

Non-profit 501(c)(3) Founded 2006

http://modelinginstruction.org/

An **Engine** for **STEM education reform:**
developed over last 30 yrs. in several overlapping phases:

I. *Research*: Design and Testing of ***Modeling Pedagogy*** (1980-1990)

II. ***Curriculum and Workshop*** *Development* (Physics: 1990-2000)
(Physical science, chemistry, biology: 2000-present)

III. *Dissemination and* ***Community building*** (1994-present)
involving 5000+ STEM teachers nationwide
2600 still subscribe to the daily Modeling Physics listserv

IV. ***Institutionalization*** in a national community of STEM teachers to drive rapid, comprehensive, high-quality reform (in progress)

V. ***Sustained R&D*** with the **STEM Modeling Institute (SMI)** (proposed)

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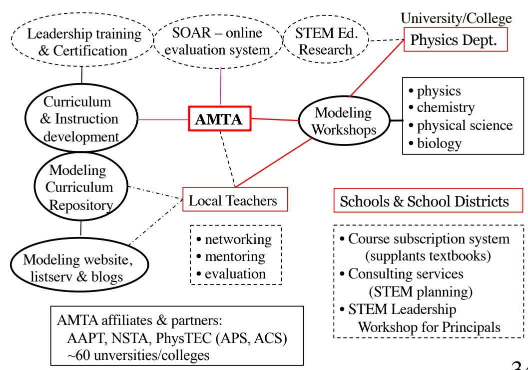
AMTA activities and services (support from SMI)

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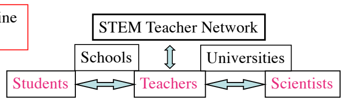
STEM Engine for reform:




**I. Cultivates teacher expertise.** To reach expert level
- requires 10 yrs. *deliberate practice* for scientists
- Teachers need equivalent *professional development*

**II. Provides access to resources** enriching STEM experience
- Summer programs, extra-curricular science & engineering
- University Outreach programs

**III. Paves a student pathway** to STEM disciplines in college
- connects high school students to university science

**Takeaway:** *Expanding the community of scientists to include STEM teachers as valued colleagues* will increase their credibility, expertise and impact!

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SUMMARY:

Scientists explore the physical world for REPRODUCIBLE PATTERNS, which they represent by MODELS and organize into THEORIES according to LAWS.

*SCIENCE is the name – MODELING is the game!*

*Now for the test:*

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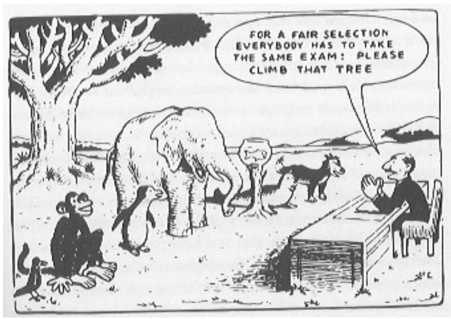


## References


Ernst Cassirer (1953), *The Philosophy of Symbolic Forms*. Yale U. Press: New Haven. Four volumes.

S. Goldman, A. Graesser & P. van den Broek (eds.), *Narrative Comprehension, Causality and Coherence*, Lawrence Erlbaum, London (1999).

David Hestenes (2008). Modeling Theory for Math & Science Education. In D. Lesh (ed.) *Mathematical Modeling ICTMA-13: Education and Design Sciences* (U. Indiana) http://geocalc.clas.asu.edu/html/Modeling.html

Bruce Sherin (2006). Common sense clarified: Intuitive knowledge and its role in physics expertise. *Journal of Research in Science Teaching* **43**: 535–555.

David Tall & Schlomo Vinner (1981) Concept Image and Concept definition in mathematics. Educational Studies in Mathematics **12**: 151-169.


A crucial distinction: **Mental models vs. Conceptual models**